

**Exercise 83**

Evaluate  $\lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1}$ .

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**Solution**

Evaluate the function at several values of  $x$  close to 1 to determine the limit.

$$\text{At } x = 0.9: \frac{0.9^{1000} - 1}{0.9 - 1} = 10$$

$$\text{At } x = 0.99: \frac{0.99^{1000} - 1}{0.99 - 1} \approx 99.9957$$

$$\text{At } x = 0.999: \frac{0.999^{1000} - 1}{0.999 - 1} \approx 632.305$$

$$\text{At } x = 0.9999: \frac{0.9999^{1000} - 1}{0.9999 - 1} \approx 951.671$$

$$\text{At } x = 0.99999: \frac{0.99999^{1000} - 1}{0.99999 - 1} \approx 995.022$$

$$\text{At } x = 0.999999: \frac{0.999999^{1000} - 1}{0.999999 - 1} \approx 999.501$$

$$\text{At } x = 1.000001: \frac{1.000001^{1000} - 1}{1.000001 - 1} \approx 1000.5$$

$$\text{At } x = 1.00001: \frac{1.00001^{1000} - 1}{1.00001 - 1} \approx 1005.01$$

$$\text{At } x = 1.0001: \frac{1.0001^{1000} - 1}{1.0001 - 1} \approx 1051.65$$

$$\text{At } x = 1.001: \frac{1.001^{1000} - 1}{1.001 - 1} \approx 1716.92$$

$$\text{At } x = 1.01: \frac{1.01^{1000} - 1}{1.01 - 1} \approx 2.10 \times 10^6$$

$$\text{At } x = 1.1: \frac{1.1^{1000} - 1}{1.1 - 1} \approx 2.47 \times 10^{42}$$

Therefore,

$$\lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1} = 1000.$$